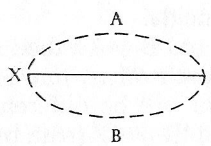
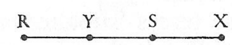
16. A Study in the Calculus of Real Addition (after 1690)

Definition 1. Those terms are 'the same' or 'coincident' of which either can be substituted for the other wherever we please without loss of truth—for example, 'triangle' and 'trilateral'. For in all the propositions about the 'triangle' proved by Euclid, 'trilateral' can be substituted without loss of truth, and conversely. *G vii. 236*



'A = B' means that A and B are the same. Thus, we may say of the straight line XY and the straight line YX that YX = XY; or, that the shortest paths of something moving from X to Y and from Y to X coincide.

Definition 2. Those terms are 'different' which are not the same, or, in which substitution sometimes does not hold. Such terms are 'circle' and 'triangle', also 'square' (namely, the perfect square; for so geometers always understand it) and 'equilateral quadrangle'; for the latter can be said of a rhombus, of which, however, it cannot be said that it is a square.



'A $\neq$ B' means that A and B are different, such as the straight lines XY and RS.

Proposition 1. If A = B, then B = A. If any term is the same as another, then the other will be the same as it. For A = B (by hypothesis), therefore (by def. 1) in the proposition 'A = B', which is true by hypothesis, B can be substituted for A and A for B; therefore the result will be B = A.

Proposition 2. If A $\neq$ B, then B $\neq$ A. If any term is different from another, that other will be different from it; otherwise we should have B = A, and therefore (by the preceding proposition) A = B, which is contrary to the hypothesis.

Proposition 3. If A = B and B = C, then A = C. Those terms which are the same as a single third term are the same as each other. For if, in the proposition 'A = B' (true by hypothesis), C is substituted for B (by def. 1, since B = C), the resulting proposition will be true.

Corollary. If A = B and B = C and C = D, then A = D; and so on. For A = B = C, therefore A = C (by the above proposition); again, A = C = D; therefore (by the above proposition) A = D.

G vii. 237 Therefore, since equals are the same in magnitude, the consequence is that things which are equal to a third thing are equal to each other. To construct an equilateral triangle Euclid makes any side equal to the base, the consequence of which is that they are equal to each other. If anything is moved in a circle, it only has to be shown that the paths of two proximate periods, i.e. of returns to the same point, always coincide for it to be concluded that the paths of any periods coincide.

Proposition 4. If A = B and B ≠ C, then A ≠ C. If, of two terms which are the same as each other, one is different from a third term, then the other also will be different from that third term. For if, in the proposition 'B ≠ C' (true by hypothesis), A is substituted for B, the result will be (by def. 1, since A = B) that the proposition 'A ≠ C' is true.

Definition 3. That A 'is in' L, or, that L 'contains' A, is the same as that L is assumed to be coincident with several terms taken together, among which is A.

Definition 4. All those terms in which there is whatever is in L will together be called 'components' in respect of L, which is 'composed' or 'constituted'.

'B ⊕ N = L' means that B is in L, or, that L contains B, and that B and N together constitute or compose L. The same holds for a larger number of terms.

Definition 5. I call those terms 'subalternants' of which one is in the other, such as A and B, whether A is in B or B is in A.

Definition 6. I call those terms 'disparate' of which neither is in the other.

Axiom 1. B ⊕ N = N ⊕ B, or, transposition makes no difference here.

Postulate 1. Given any term, some term can be assumed which is different from it, and, if one pleases, disparate, i.e. such that the one is not in the other.

Postulate 2. Any plurality of terms, such as A and B, can be taken together to compose one term, A ⊕ B, or, L.

Axiom 2. A ⊕ A = A. If nothing new is added, nothing new is made; i.e. repetition changes nothing here. (For although four coins and another four are eight coins, four coins and the same four already counted are not.)

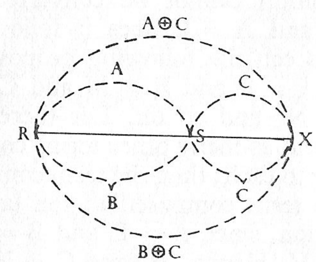
Proposition 5. If A is in B, and A = C, then C is in B. The coincident of an inexistent is an inexistent. For in the proposition 'A is in B' (true by hypothesis), the substitution of C for A (by the definition of coincidents, def. 1, since A = C by hypothesis) has the result that C is in B.

Proposition 6. If C is in B, and A = B, then C will be in A. What is in one of two coincidents is also in the other. For in the proposition 'C is in B', the substitution of A for C (since A = C) has the result that A is in B (this is the converse of the preceding proposition).[1]

Proposition 7. A is in A. Every term is in itself. For A is in A ⊕ A (by the definition of 'inexistent', i.e. by def. 3), and A ⊕ A = A *G vii. 238* (by axiom 2). Therefore (by prop. 6) A is in A.

Proposition 8. A is in B, if A = B. One of two coincidents is in the other. This is evident from the preceding proposition. For A is in A (by the preceding proposition), that is (by hypothesis) it is in B.

Proposition 9. If A = B, A ⊕ C = B ⊕ C. If coincidents are added to the same term, coincidents result. For if, in the proposition 'A ⊕ C = A ⊕ C' (true in itself) you substitute for A in one place its coincident B (by def. 1), the result will be A ⊕ C = B ⊕ C.



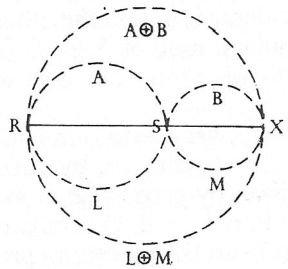
A 'triangle' }
B 'trilateral' } coincide

A ⊕ C 'equilateral triangle' }
B ⊕ C 'equilateral trilateral' } coincide

Note. This proposition cannot be converted, much less the two which follow. A method of finding an instance of this will be shown below, in the problem which constitutes proposition 23.

[1] The text has been followed here, but it will be noticed that Leibniz does not prove the proposition enunciated. In his proof he assumes, not that A = B, but that A = C, and he gives what is in effect another proof of proposition 5. The proof of proposition 6 should have run: 'In the proposition "C is in B", the substitution of A for B (since A = B) has the result that C is in A' (cf. Kneale, p. 341).

Proposition 10. If A = L and B = M, then A ⊕ B = L ⊕ M. If coincidents are added to coincidents, coincidents result. For since B = M, then (by the preceding proposition) A ⊕ B = A ⊕ M, and putting L for the second A (since A = L, by hypothesis) the result will be A ⊕ B = L ⊕ M.

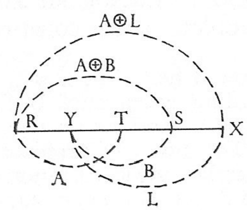


A 'triangle', L 'trilateral' coincide; B 'regular', M 'most capacious of equally many-sided figures with equal perimeters' coincide. 'Regular triangle' and 'most capacious of trilaterals making equal peripheries out of three sides' coincide.

Note. This proposition cannot be converted; for not even if A ⊕ B = L ⊕ M and A = L does it follow immediately that B = M. Much less can the following proposition be converted.

Proposition 11. If A = L and B = M and C = N, then A ⊕ B ⊕ C = L ⊕ M ⊕ N; and so on. Let there be assumed any number of terms, and as many other terms coincident with them, each corresponding to each; then the term composed of the former coincides with the term composed of the latter. For (from the preceding proposition, since A = L and B = M) the result will be A ⊕ B = L ⊕ M. Hence, because C = N, the result will be (again by the preceding proposition) A ⊕ B ⊕ C = L ⊕ M ⊕ N. *G vii. 239*

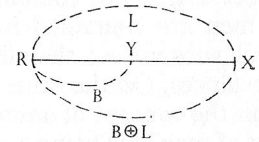
Proposition 12. If B is in L, then A ⊕ B will be in A ⊕ L. If the same term is added to content and to container, the result of the former operation is in the result of the latter. For let L = B ⊕ N (by the definition of 'inexistent'); A ⊕ B is also in B ⊕ N ⊕ A (by the same), that is, in L ⊕ A.



B 'equilateral', L 'regular', A 'quadrilateral'. 'Equilateral' is in, i.e. is ascribed to 'regular'. Therefore 'equilateral quadrilateral' is in 'regular quadrilateral', i.e. in 'perfect square'. YS is in RX. Therefore RT ⊕ YS, i.e. RS, is in RT ⊕ RX, i.e. in RX.

Note. This proposition cannot be converted; for not even if A ⊕ B is in A ⊕ L does it follow that B is in L.

Proposition 13. If L ⊕ B = L, then B will be in L. If any term does not become another when another is added to it, the added term is in it. For B is in L ⊕ B (by the definition of 'inexistent'), and L ⊕ B = L (by hypothesis), therefore (by prop. 6) B is in L.



RY ⊕ RX = RX, therefore RY is in RX.

RY is in RX, therefore RY ⊕ RX = RX.

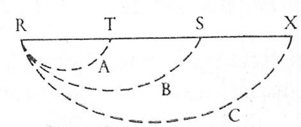
Let L be 'parallelogram' (of which any side is parallel to some side), B be 'quadrilateral'.

'Quadrilateral parallelogram' is the same as 'parallelogram', therefore 'being quadrilateral' is in 'parallelogram'.

Conversely, 'being quadrilateral' is in 'parallelogram', therefore 'quadrilateral parallelogram' is the same as 'parallelogram'.

Proposition 14. If B is in L, then L ⊕ B = L. Subalternants compose nothing new; i.e. if any term which is in another is added to it, it does not make anything which is different from that other. This is the converse of the preceding proposition. If B is in L, then (by the definition of 'inexistent') L = B ⊕ P; therefore (by prop. 9) L ⊕ B = B ⊕ P ⊕ B, that is (by axiom 2) = B ⊕ P, which (by hypothesis) = L.

Proposition 15. If A is in B and B is in C, then A is in C. A content of a content is a content of the container. For A is in B (by hypothesis), therefore A ⊕ L = B (by the definition of 'inexistent'). *G vii. 240* Similarly, because B is in C, B ⊕ M = C. Putting, in this assertion, A ⊕ L for B (which we have shown to coincide), the result will be A ⊕ L ⊕ M = C. Therefore (by the definition of 'inexistent') A is in C.



RT is in RS and RS is in RX, therefore RT is in RX.

A 'quadrilateral', B 'parallelogram', C 'rectangle'.

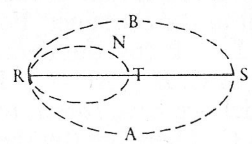
'Being quadrilateral' is in 'parallelogram', and 'being a parallelogram' is in 'rectangle' (i.e. a figure every angle of which is a right angle). Therefore 'being quadrilateral' is in 'rectangle'. These can be inverted, if instead of concepts considered in themselves we consider the individuals [*singularia*] comprehended under a concept; A can be a rectangle, B a parallelogram, C a quadrilateral. For all rectangles are comprehended in the number of parallelograms, and all parallelograms in the number of quadrilaterals; therefore all rectangles are contained in quadrilaterals. In the same way, all men are contained in all animals, and all animals in all corporeal substances; therefore all men are contained in corporeal substances. On the other hand, the concept of corporeal substance is in the concept of animal and the concept of animal is in the concept of man; for being a man contains being an animal.

Note. This proposition cannot be converted, much less the one which follows.

Corollary. If $A \oplus N$ is in B, then N is in B. For N is in $A \oplus N$ (by the definition of 'inexistent').

Proposition 16. If A is in B and B is in C and C is in D, then A is in D; and so on. A content of what is contained by a content is a content of the container. For if A is in B and B is in C, A is also in C (by the preceding proposition). Hence if C is in D, then (again by the preceding proposition) A will also be in D.

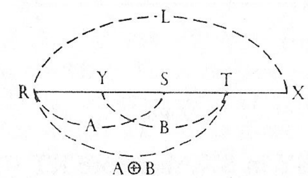
Proposition 17. If A is in B and B is in A, then $A = B$. Terms which contain each other coincide. For if A is in B, then $A \oplus N = B$ (by the definition of 'inexistent'). Now, B is in A (by hypothesis), therefore $A \oplus N$ is in A (by prop. 5). Therefore (by the corollary to prop. 15) N is also in A; therefore (by prop. 14) $A = A \oplus N$, i.e. $A = B$.



RT, N; RS, A; $SR \oplus RT$, B.

G vii. 241 'Being trilateral' is in 'triangle', and 'being a triangle' is in 'trilateral'. Therefore 'triangle' and 'trilateral' coincide. So also with 'being omniscient' and 'being omnipotent'.

Proposition 18. If A is in L and B is in L, then $A \oplus B$ will be in L. What is composed of two terms which are inexistent in the same term, is in that same term. For because A is in L (by hypothesis), it can be seen that $A \oplus M = L$ (by the definition of 'inexistent'). Similarly, because B is in L it can be seen that $B \oplus N = L$. Putting these together we have (by prop. 10) $A \oplus M \oplus B \oplus N = L \oplus L$. Therefore (by axiom 2)[1] $A \oplus M \oplus B \oplus N = L$. Therefore (by the definition of 'inexistent') $A \oplus B$ is in L.



RYS is in RX.

YST is in RX.

Therefore RT is in RX.

A 'equiangular', B 'equilateral', $A \oplus B$ 'equiangular equilateral', i.e. 'regular', L 'square'. 'Equiangular' is in 'square', 'equilateral' is in 'square', therefore 'regular' is in 'square'.

Proposition 19. If A is in L and B is in L and C is in L, then $A \oplus B \oplus C$ will be in L; and so on. Or, in general: if each of a number of terms taken severally is in something, so also is that which is composed of them. For $A \oplus B$ will be in L (by the preceding proposition); now, C is in L (by hypothesis), therefore (again by the preceding proposition) $A \oplus B \oplus C$ is in L.

Note. It is evident that these two, and similar propositions, can be converted. For if $A \oplus B = L$,[2] it is evident from the definition of 'inexistents'[3] that A is in L and B is in L. Also, if $A \oplus B \oplus C = L$, it is evident that A is in L, and B is in L, and C is in L; also that $A \oplus B$ is in L, that $A \oplus C$ is in L, and that $B \oplus C$ is in L; and so on.

Proposition 20. If A is in M and B is in N, then $A \oplus B$ will be in $M \oplus N$. If the former term of one pair of terms is in the latter, and the former term of another pair of terms is in the latter term of that pair, then the term composed of the two former terms is in the term composed of the two latter terms. For A is in M (by

[1] The text has 'Axiom 5'.

[2] C. I. Lewis (p. 301) points out that Leibniz should in consistency have said that $A \oplus B$ is in L, and similarly that $A \oplus B \oplus C$ is in L.

[3] The text has 'existentium'.

hypothesis), and M is in $M \oplus N$ (by the definition of 'inexistent'). Therefore (by prop. 15) A is in $M \oplus N$. Similarly, because B is in N, and N is in $M \oplus N$, B will be in $M \oplus N$. Now, if A is in $M \oplus N$ and B is in $M \oplus N$, then (by prop. 18) $A \oplus B$ will be in $M \oplus N$.

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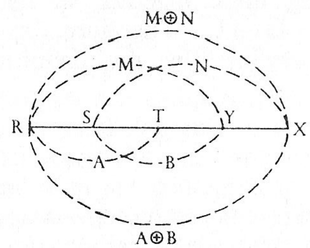


RT is in RY and SY in SX, therefore $RT \oplus SY$, i.e. RY, is in $RY \oplus SX$, i.e. in RX.[1]

Let A be 'quadrilateral', B be 'equiangular'; $A \oplus B$ will be 'rectangle'. Let M be 'parallelogram', N be 'regular'; $M \oplus N$ will be 'square'. Now, 'quadrilateral' is in 'parallelogram' and 'equiangular' is in 'regular'; therefore 'rectangle' (i.e. 'equiangular quadrilateral') is in 'regular parallelogram', i.e. 'square'.

Note. This proposition cannot be converted. Even granting that A is in M and that $A \oplus B$ is in $M \oplus N$, yet it does not follow immediately that B is in N. For it can happen that both A and B are in M, and also that some terms which are in B are in M, but the rest are in N. Much less, therefore, can the following proposition and any like it be converted.

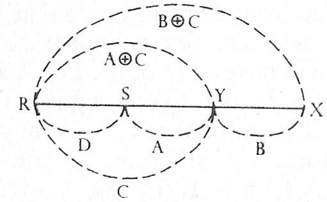
Proposition 21. If A is in M and B is in N and C is in P, $A \oplus B \oplus C$ will be in $M \oplus N \oplus P$; and so on. That which is composed of contents is in that which is composed of the containers. For because A is in M and B is in N, $A \oplus B$ will be in $M \oplus N$ (by the preceding proposition). Now, C is in P, therefore (again by the preceding proposition) $A \oplus B \oplus C$ is in $M \oplus N \oplus P$.

Proposition 22. Given two disparate terms, A and B, to find a third term, C, which is different from them and which together with them makes up the subalternants $A \oplus C$ and $B \oplus C$: that is, that although neither of A and B is in the other, yet one of $A \oplus C$ and $B \oplus C$ is in the other.

Solution. If we want $A \oplus C$ to be in $B \oplus C$, although A is not in B, this can be done as follows. Let there be assumed (by post. 1)

[1] The text has 'RT est in RY et ST in SX, ergo RT $\oplus$ ST seu RY est in RY $\oplus$ SY seu in RY'. In the translation, this has been corrected from the diagram and from the example which follows.

some term, D, of any kind provided that it is not in A, and (by post. 2) let $A \oplus D = C$; then we shall have done what is required.



For $A \oplus C = A \oplus A \oplus D$[1] (by the construction) $= A \oplus D$ (by axiom 2). Similarly $B \oplus C = B \oplus A \oplus D$ (by the construction). Now $A \oplus D$ is in $B \oplus A \oplus D$ (by def. 3). Therefore $A \oplus C$ is in $B \oplus C$; which was to be done.

SY and YX are disparate. Let $RS \oplus SY = YR$, $SY \oplus YR$ will be in $XY \oplus YR$.

Let A be 'equilateral', B 'parallelogram', D 'equiangular', C 'equiangular equilateral', i.e. 'regular'; here it is evident that, although 'equilateral' and 'parallelogram' are disparate, such that the one is not in the other, yet 'regular equilateral' is in 'regular parallelogram', i.e. 'square'. But, you will say, the construction described above in the problem will not succeed in all cases. For example, let A be 'trilateral' and B 'quadrilateral'; a concept cannot be found in which there are at the same time both A and B, and therefore there is not a $B \oplus C$ in which there is $A \oplus C$, because A and B are incompatible. I reply that our general construction depends upon the second postulate, in which is contained the proposition that any term can be compounded with any term. Thus, God, soul, body, point and heat compose an aggregate of these five things. So 'quadrilateral' and 'triangle' can also be compounded, and the problem is solved. For let it be assumed that D is anything which is not contained in 'trilateral', such as 'circle'; $A \oplus D$ will be 'trilateral and circle', which may be called C. Now, $C \oplus A$ is nothing but 'trilateral and circle' again, and this is in $C \oplus B$, that is, in 'trilateral, circle and quadrilateral'. But if anyone wishes to apply this general calculus of compositions of any sort to a special manner of composition—e.g. if he wants 'trilateral', 'circle' and 'quadrilateral' not only to compose one aggregate, but also to be in the same subject, each concept at the same time—then he must see if they are compatible. Thus, unmoved straight lines at a distance from each other can be

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[1] The text has L in the proof where the diagram and the solution have D. D has been substituted in the translation.

taken together to compose one aggregate, but not to compose one continuum.

Proposition 23. Given two disparate terms, A and B, to find a third term, C, different from them and such that $A \oplus B = A \oplus C$.[1]

Solution. Let it be assumed (by postulate 2) that $C = A \oplus B$, and what is desired will have been done. For A and B are disparate (by hypothesis), that is (by def. 6) one is not in the other, therefore (by prop. 13) it is not possible that $C = A$, or that $C = B$. These three terms, therefore, are different, as the problem requires. Further, $A \oplus C = A \oplus A \oplus B$ (by the construction), that is (by axiom 2) $= A \oplus B$. Therefore $A \oplus C = A \oplus B$; which is what was to be done.

Proposition 24. To find several terms which are different, each to each, as many as shall be desired, such that from them there cannot be composed a term which is new, i.e. different from any of them.

Solution. Let there be assumed (by post. 1) any terms whatever and of any number, which are different from each other, A, B, C, D; of these (by post. 2) let $A \oplus B = M$, $M \oplus C = N$, $N \oplus D = P$. I assert that A, B, M, N, P are the required terms. For M (by the construction) is made from A and B; further, A or B is in M, M is in N, and N is in P. Therefore (by prop. 16) any one of the former is in any one of the latter. Now, if you compound any two of these with each other, nothing new is constituted. For if
G vii. you compound the same term with itself, nothing new is formed;
244 $L \oplus L = L$ (by axiom 2).[2] If you compound one term with another, you compound the former with the latter, and therefore a term which is inexistent with the term which contains it, as $L \oplus N$; but $L \oplus N = N$ (by proposition 14).[3] If you compound three terms, such as $L \oplus N \oplus P$, you compound the pair of terms $L \oplus N$ with the one term P. But the pair $L \oplus N$ of themselves constitute nothing new, but from them (as we have already shown) there comes one term, namely the latter term N. Therefore to compound the pair of terms $L \oplus N$ with the one term P is the same as compounding the one term N with the one term P, which we have already shown to constitute nothing new. Therefore the pair of terms together with the one term—i.e. the three together—constitute nothing new. And so on, in the case of more terms. Which is what was to be done.

[1] The clause 'and such that . . .' is not in the text, but is demanded by the proof and by a reference in the note to prop. 9. Cf. Lewis, p. 302.
[2] The number of the axiom is not given in the text.
[3] The number of the proposition is not given in the text.

Note. It would have been sufficient to assume terms which exist successively in each other, as M, N, P, &c.; and indeed this will hold if in our construction we put A = Nothing, with the result that $B = M$. However, the solution which has been given extends somewhat more widely. Indeed, these problems can be solved in yet other ways; but to exhibit all possible solutions of the problems, i.e. to prove that no other methods are possible, needs the prior proof of several other propositions. For example, the five things A, B, C, D and E can be arranged so that nothing new can be compounded from them only in the following ways: first, if A is in B and B in C and C in D and D in E; second, if $A \oplus B = C$ and C is in D and D in E; third, if $A \oplus B = C$ and A is in D and $B \oplus D = E$. The five concepts 'equiangular', A, 'equilateral', B, 'regular', C, 'rectangle', D, and 'square', E, are related in the third or last way. No new term can be compounded from these which does not coincide with them; for 'equiangular equilateral' coincides with 'regular', 'equiangular' is in 'rectangle', and 'equilateral rectangle' coincides with 'square'. Hence 'equiangular regular' is the same as 'regular' and 'equilateral regular' also, and 'equiangular rectangle' is 'rectangle' and 'regular rectangle' is 'square'.

Note to definitions 3, 4, 5 and 6. We say that the concept of the genus is in the concept of the species, the individuals of the species in the individuals of the genus; a part in the whole, and the indivisible in the continuum—such as a point in a line, even though a point is not a part of a line. Thus, the concept of an affection or predicate is in the concept of the subject. In general, this consideration extends very widely. We also say that inexistents are contained in those terms in which they are. Nor does it matter here, with regard to this general concept, how those terms which are in something are related to each other or to the container. So our proofs hold even of those terms which compose something distributively, as all species together compose the genus. Further, all the inexistents which are sufficient to constitute a container, or,
in which there are all the terms which are in the container, are *G vii.*
said to compose the container itself. For example, $A \oplus B$ will *245*
be said to 'compose' L if A, B and L stand for the straight lines RS, YX and RX, for $RS \oplus YX = RX$. In the same way, $RS \oplus SX = RX$. Such parts, which complete a whole, I customarily term 'co-integrants'[1], especially if they have no common part, such that they can be called 'co-members', like RS and RX. From this is it evident that the same term can be composed in

[1] Cf. *Specimen Geometriae Luciferae*, G. *M.* vii. 284.

many ways, if the terms of which it is composed are again composite; and further, that if they can be analysed to infinity, then the variations of composition are infinite. Therefore the whole of synthesis and analysis depends on the principles laid down here. Further, if the terms which are in something are homogeneous with that in which they are contained, they are called 'parts' and the container is called a 'whole'. If any two parts are so related that a third thing can be found which has a part common to the one and a part common to the other, that which is composed of them is a continuum. From this it is evident in what way one consideration rises gradually from another. Further, I call 'subalternants' those of which one is in the other, as a species in a genus, or the straight line RS in the straight line RX. I call them 'disparate' when the case is different; such as the straight lines RS and YX, two species of the same genus, a perfect and an imperfect metal, and also the members of different divisions of the same whole, which have something in common. For example, if you divide 'metal' into 'perfect' and 'imperfect', and again into 'soluble in *aqua fortis*' and 'insoluble in *aqua fortis*', it is evident that 'metal insoluble in *aqua fortis*' and 'perfect metal' are two disparate terms, and that there is a perfect metal (i.e. which is fulminable, remaining in the cupel)[1] which is yet soluble in *aqua fortis*, such as silver; and that on the other hand there is an imperfect metal which is insoluble in *aqua fortis*, such as tin.

Note to axioms 1 and 2. As general algebra [*speciosa generalis*] is merely the representation and treatment of combinations by signs, and as various laws of combination can be discovered, the result of this is that various methods of computation arise. Here, however, no account is taken of the variation which consists in a change of order alone, and AB is the same for us as BA. Next, no account is taken here of repetition; i.e. AA is the same for us as A. Consequently, whenever these laws are observed, the present calculus can be applied. It is evident that this is observed in the composition of absolute concepts, where no account is taken of order or of repetition. Thus, it is the same to say 'hot and bright' as to say 'bright and hot', and to speak of 'hot fire' or 'white milk', with the poets, is a pleonasm; 'white milk' is simply 'milk', and 'rational man'—i.e. 'rational animal which is rational'—is simply 'rational

[1] Reading, with Lewis, 'cupella' for 'capella'. The reference is probably to 'cupellation', a process by which impurities are removed from impure precious metal, the purified metal remaining in the crucible (see, e.g., E. J. Holmyard, *Alchemy* (London, 1957), p. 41). 'Fulmination', as a metallurgical term, means 'becoming suddenly bright and uniform in colour'.

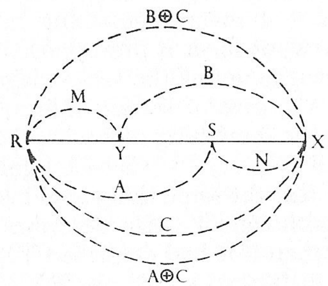
animal'. It is the same when certain determinate things are said to exist in things: real addition of the same thing is vain repetition. When two and two are said to make four, the latter two must be different from the former. If they were the same, nothing new would result; it would be just as if, for a joke, I wanted to make six eggs out of three by first counting three eggs, then taking away one and counting the remaining two, and finally taking one away again and counting the remaining one. But in the calculus of numbers and magnitudes, A, B or other signs do not stand for a certain thing, but for any thing of the same number of congruent parts. For any two feet are signified by 2, if a foot is the unit or measure, whence $2 + 2$ makes something new, 4, and 3 by 3 makes something new, 9; for it is presupposed that what are used are always different (though of the same magnitude). The situation is different in the case of certain things, for example lines. Let it be assumed that something moveable describes the straight line $RY \oplus YX = RYX$, or, $P \oplus B = L$, going from R to X. Then let us assume that the same thing goes back from X to Y and stays there; then, although it twice describes YX or B, it produces nothing else than if it had described YX once. So '$L \oplus B$' is the same as L, i.e. '$P \oplus B \oplus B$'; or, '$RY \oplus YX \oplus XY$' is the same as '$RY \oplus YX$'. This caution is of great importance in estimating the magnitude of things which are generated by the magnitude of the motion of those things which generate[1] or describe. For care must be taken that, in describing, one thing does not choose as its own path the track of another, or that one part of the describer does not succeed to the place of another; or there must be a subtraction, so that there is no reduplication. It is also evident from this that, according to the concept which we are using here, components can by their magnitudes constitute a magnitude which is greater than that of the thing which they compose. Hence the composition of things and of magnitudes differs widely. For example, if a straight line L, or RX, has two parts, A, or RS, and B, or YX, either of which is greater than half of RX—e.g. if RX is five feet, RS four feet and YX three feet—it is evident that the magnitudes of these parts will constitute a magnitude of seven feet, greater than the magnitude of the whole. Yet the straight lines RS and YX compose nothing other than RX, i.e. $RS \oplus YX = RX$. This is why I here designate this real addition by $\oplus$, as the addition of magnitudes is designated by $+$. Finally: when, in real addition, one is concerned with the actual generation of things, it makes a great difference what the order is—for the foundations are laid before the house is built. But in the mental formation of

G vii. 246

[1] Reading, with Lewis, 'generant' for 'generantur'.

things the result is the same, no matter which ingredient we consider first (although one method of consideration may be more useful than another), so the order does not make any change in the thing which is produced. In due course order also will be considered; for the moment, however, 'RY $\oplus$ YS $\oplus$ SX' is the same as 'YS $\oplus$ RY $\oplus$ SX'.

Note to proposition 24. Given that RS and YX are different, indeed disparate, so that neither is in the other, let RS $\oplus$ YX = RX;
G vii. then 'RS $\oplus$ RX' will be the same as 'YX $\oplus$ RX'. For, in the case
247 of concepts, it is always the straight line RX which is composed.



Let A be 'parallelogram', B 'equiangular' (which are disparate) and let C be A $\oplus$ B, i.e. 'rectangle'. Then 'rectangular parallelogram' will be the same as 'equiangular rectangle', for each of the two is simply 'rectangle'. Generally, let Maevius be A, Titius B, and the pair of men composed of the two be C; then Maevius with this pair will be the same as Titius with this pair, for in neither case does anything result other than the pair itself. Still another solution can be given, which is more elegant but more restricted, if A and B have something in common, and this is given, and so what is peculiar to each is also given. Let it be, therefore, that M is peculiar to A and N is peculiar to B; let M $\oplus$ N = D, and let P be common to each; I assert that A $\oplus$ D = B $\oplus$ D. For since A = P $\oplus$ M and B = P $\oplus$ N, A $\oplus$ D = P $\oplus$ M $\oplus$ N, and again B $\oplus$ D = P $\oplus$ M $\oplus$ N.